Math 72 chapter 9 } Logarithms
Math 62 chapter 11

section 1: Composition of functions
Inverse Functions

section 2: Exponential functions
including base e

section 3: Logarithmic functions
including common logs
and natural logs

section 4: Properties of Logs
including change-of-base formula

section 5: Solving Exponential Equations
Solving Logarithmic Equations

Logarithms are a key component of the course and will appear multiple times on the final exam.

Logarithms

Objectives

1) Review the algebra of functions
   - addition
   - subtraction
   - multiplication
   - division

2) Find the composition of two functions

3) Identify which functions are composed to get a given function.

4) Observe that some functions "undo" each other when composed, while others do not.

5) Use inverse notation $f^{-1}(x)$ and speak it correctly.

6) Find the inverse of a function
   - from a list of ordered pairs
   - algebraically.
   - Is the result a function?

7) Use the horizontal line test to determine if the inverse will be a function.

8) Identify if a function is an invertible function

9) Graph functions and their inverses

10) Use algebra to show that two functions are inverses

Math 70 12.1 Algebra of Functions

<u>Overview of Chapter 12</u>	
12.1	Function Composition
	Inverse Functions [Functions that "un-do" each other when composed]
12.2	Exponential Functions
12.3	Logarithmic Functions [Inverse functions (12.1) of Exponential Functions (12.2)]
12.4	Properties of Logarithms [Unexpected traits of Logarithmic functions (12.3)]
12.5	Natural logs, and Change of Base [Special logarithmic functions(12.3) and GC]
12.6 & 12.7	Solving Exponential and Logarithmic Equations and Applications
*This chapter builds one section on the next, layering complicated concepts.	

Objectives

1) Find a new function which is composition $(f \circ g)(x)$ of two given functions.
2) Review 5.9: the sum $(f+g)(x)$, difference $(f-g)(x)$, product $(f \cdot g)(x)$, quotient $(f/g)(x)$
3) Recognize function notation and notation for the names of these functions.
4) Practice negative and positive exponents on common bases, in preparation for 12.2.

Practice and Examples

✓1) Given $f(x)=3x^2+4x+1$ and $g(x)=2x-5$, find:

a) $(f+g)(x)$
b) $(f-g)(x)$
c) $(f \cdot g)(x)$
d) $(f/g)(x)$
e) $(g \circ f)(x)$
f) $(f \circ g)(x)$

2) Given $\begin{cases} f(-1)=4 & g(-1)=-4 \\ f(0)=5 & g(0)=-3 \\ f(2)=7 & g(2)=-1 \\ f(7)=1 & g(7)=9 \end{cases}$, find

✓ a) $(f+g)(2)$
✓ b) $(f-g)(0)$
c) $(f \cdot g)(7)$
✓ d) $(f \cdot g)(0)$
e) $(f/g)(0)$
✓ f) $(g/f)(0)$
✓ g) $(g \circ f)(2)$
✓ h) $(f \circ g)(2)$

<u>Recall:</u> Function Notation $f(x)$ means "f of x" where f is the name of the function and (x) tells the variable used in the function.

In 9.1 we will write $(f+g)(x)$, spoken "f plus g of x", which means the name of the function is "f plus g" and the variable being used is x.

<u>CAUTION:</u> "of x" does <u>not</u> mean "multiply by x"

---

① Given $f(x)=3x^2+4x+1$ and $g(x)=2x-5$, find

a) $(f+g)(x)$.

name of new function — function notation "of x" indicates variable used.

f+g is the name of the new function

(f+g)(x) is pronounced "f plus g, of x"

Method: Add $f(x)$ and $g(x)$. $(f+g)(x) = f(x)+g(x)$.

$= f(x)+g(x)$

$= (3x^2+4x+1)+(2x-5)$ — subst expressions for $f(x)$ & $g(x)$.

$= \boxed{3x^2+6x-4}$ — combine like terms = add.

b) $(f-g)(x)$

$= f(x)-g(x)$ — "f-g" is the name of the new function — "f minus g, of x"

$= (3x^2+4x+1)-(2x-5)$ — substitute expressions * must use ( )

$= 3x^2+4x+1-2x+5$ — distribute negative

$= \boxed{3x^2+2x+6}$ — combine like terms

c) $(f\cdot g)(x)$

Handwriting alert! $f\cdot g$ is different from $f\circ g$

$f\cdot g$: small dot = multiply

$f\circ g$: loop = compose

$= f(x) \cdot g(x)$ "f times g, of x" or simply "fg of x"

$= (3x^2+4x+1)(2x-5)$ substitute * must use ( )

$= 3x^2(2x-5) + 4x(2x-5) + 1(2x-5)$ distribute each term of first

$= 6x^3 - 15x^2 + 8x^2 - 20x + 2x - 5$

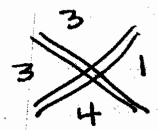
$= 6x^3 \quad x^2 - 18x - 5$ combine

d) $(f/g)(x)$ "f divide by g, of x"

$= \dfrac{f(x)}{g(x)}$

$= \dfrac{3x^2+4x+1}{2x-5}$ factor and cancel to simplify fraction, if possible

$= \boxed{\dfrac{(3x+1)(x+1)}{(2x-5)}}$



$3x^2+3x+x+1$

$= 3x(x+1) + 1(x+1)$

$= (x+1)(3x+1)$

Putting one function value inside another is called function composition.

This is the most important skill in 9.1 because we need it to

- un-do functions (inverse functions)
- un-do exponential functions specifically (logarithms).

$f(g(x))$ is also called $(f \circ g)(x)$ or "f composed on g of x".

e) $(g \circ f)(x)$ "g composed on f, of x"

$= g(f(x))$ keep the order of the functions

$= 2(\ )-5$

replace all x's in g(x) by f(x)

$= 2(f(x)) - 5$ subst for f(x)

$= 2(3x^2+4x+1) - 5$ simplify.

$= 6x^2+8x+2-5$ distribute 2

$= \boxed{6x^2+8x-3}$ combine

f) $(f \circ g)(x)$ "f composed on g, of x"

$= f(g(x))$ keep the order of the functions

$= 3(\ )^2+4(\ )+1$

replace all x's in f(x) by g(x)

$= 3(g(x))^2+4(g(x))+1$ substitute for g(x)

$= 3(2x-5)^2+4(2x-5)+1$ simplify using order of operations

- exponents
- then multiply
- then add/subtract L→R.

$= 3\underbrace{(2x-5)(2x-5)}_{\text{exponent = FOIL}}+4(2x-5)+1$

$= 3(4x^2-20x+25)+4(2x-5)+1$ distribute

$= 12x^2-60x+75+8x-20+1$ combine

$= \boxed{12x^2-52x+56}$

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(2) If we are given that

$f(-1)=4 \qquad g(-1)=-4$
$f(0)=5 \qquad g(0)=-3$
$f(2)=7 \qquad g(2)=-1$
$f(7)=1 \qquad g(7)=9$

Find

a) $(f+g)(2)$ "f plus g of 2"
$= f(2)+g(2)$ method
$= 7+(-1)$ substitute given values
$= \boxed{6}$

If we graph $y_1 = f(x)$
$y_2 = g(x)$
$y_3 = (f+g)(x)$
and look at a table of values, we will see that, at a particular value of x, $y_3 = y_2 + y_1$ is the sum of the y-values

b) $(f-g)(0)$ "f minus g of zero"
$= f(0) - g(0)$ method
$= 5 - (-3)$ substitute
$= 5+3$
$= \boxed{8}$

c) $(f \cdot g)(7)$ "f times g of 7"
$= f(7) \cdot g(7)$ method
$= 1 \cdot 9$ substitute
$= \boxed{9}$

d) $(f \cdot g)(0)$ "f times g of zero"

$= f(0) \cdot g(0)$ method

$= (5)(-3)$ substitute

$= \boxed{-15}$

e) $(f/g)(0)$ "f divide by g of zero"

$= f(0) / g(0)$ method

$= 5/(-3)$ substitute

$= \boxed{\frac{-5}{3}}$

NOTICE: Not multiply by zero.
Yes: Evaluate at zero.

f) $(g/f)(0)$ "g divide by f of zero"

$= g(0) / f(0)$

$= -3/5$

$= \boxed{\frac{-3}{5}}$

---

g) $(g \circ f)(2)$

Find $g(f(2))$

step 1: find $f(2) = 7$
step 2: put result into g
find $g(7) = 9$
answer $\boxed{9}$

---

h) $(f \circ g)(2)$ Find $f(g(2))$

work from the inside out

step 1: find $g(2) = -1$
step 2: put this result (-1) into f
find $f(-1) = 4$
answer $= \boxed{4}$

Math 70 12.1 Inverse Functions

Objectives

1) Observe that inverse functions, when composed, "un-do" each other.
    a) Notation $f^{-1}(x)$ is pronounce "f-inverse of x".

CAUTION: This notation looks like an exponent, but it's not. Exponent would be outside: $[f(x)]^{-1} = \frac{1}{f(x)}$

CAUTION: $f^{-1}(x)$ is NOT usually the reciprocal of $f(x)$.

    b) "Un-do" means: $(f^{-1} \circ f)(x) = x$, $(f \circ f^{-1})(x) = x$

2) Find the inverse of a function.
    a) From a list of ordered pairs
    b) Algebraically
    c) Is the resulting inverse a function?
3) Determine if a function has an inverse function, AKA "is an invertible function". A function has an inverse function if:
    a) it is one-to-one: for each y value there is at most one x value.
    b) its graph passes the horizontal line test.
    c) Recall: To be a function, the graph must pass the vertical line test.

CAUTION: A graph can be one-to-one and not be a function, or vice-versa. To be an "invertible function", it must pass both the VLT and the HLT.

4) Graph functions and their inverses.
5) Use algebra to show that two functions are inverses of each other. Show that: $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$

Practice and Examples

1) Given $f(x) = 2x+3$ and $g(x) = \frac{1}{2}(x-3)$, find:
    a) $(g \circ f)(x)$

    b) $(f \circ g)(x)$

    c) $(g \circ f)(23)$

2) Complete the tables for $f(x)=2x+3$ and $g(x)=\frac{1}{2}(x-3)$

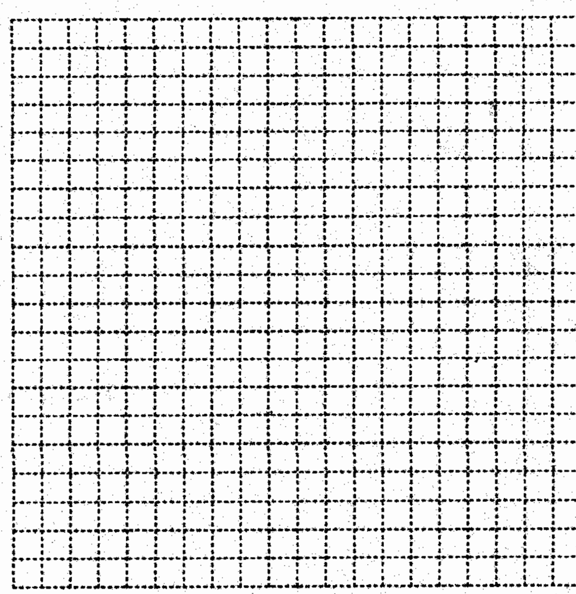
x	y=f(x)
-2	
0	
1	
52	
-1	
3	
5	
107	

x	y= g(x)
-1	
3	
5	
107	
-2	
0	
1	
52	

3) Find the inverse of the function. Graph the points of the function. Is the inverse a function?

x	y=f(x)
0	2
1	3
2	4
3	5

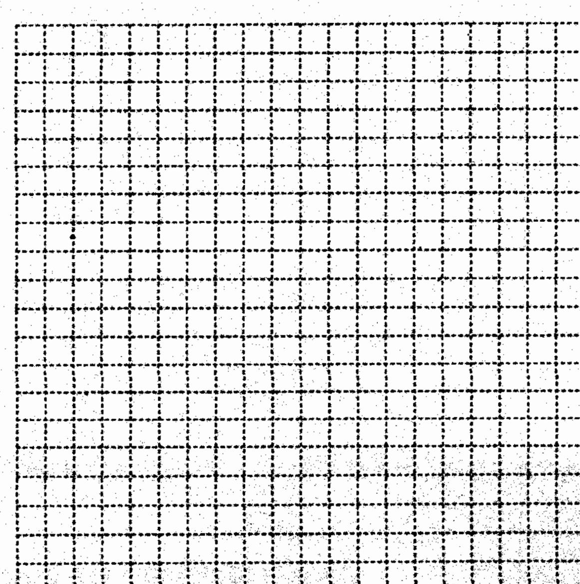
x	y



4) Find the inverse of the function. Graph the points of the function. Is the inverse a function?

x	y=f(x)
-1	2
1	3
2	4
5	3

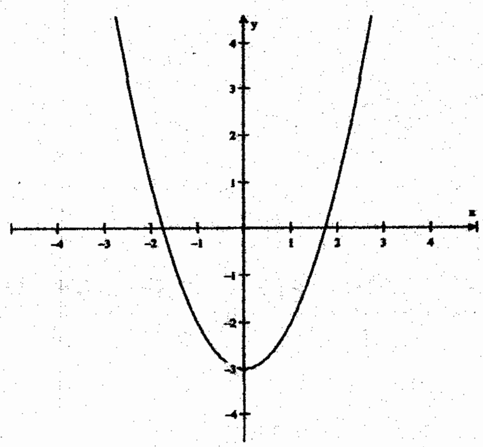
x	y



For each graph, identify

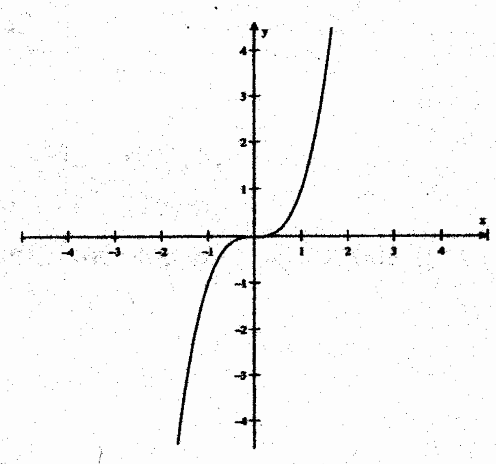
a) Is it a function?
b) Is it one-to-one?
c) Is it an invertible function?

5)



yes

6)



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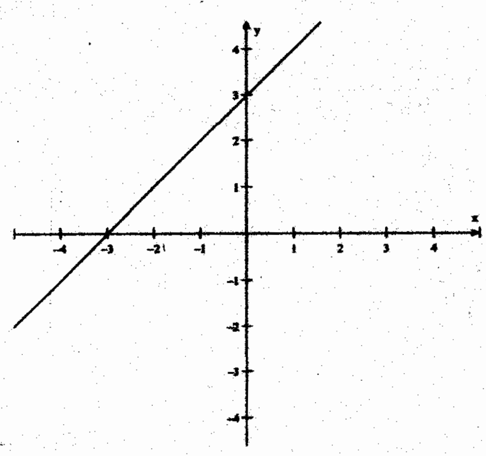
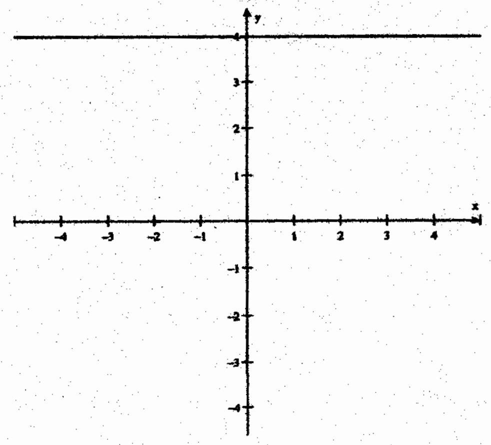
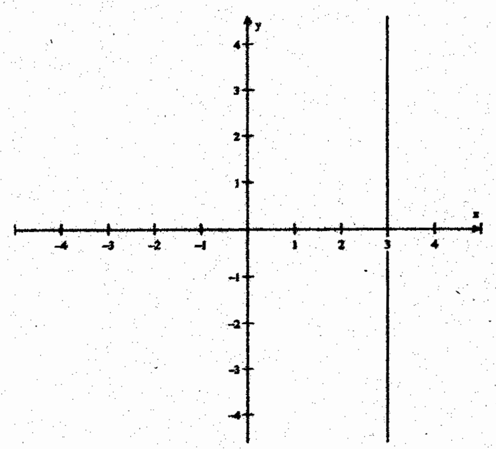


8)



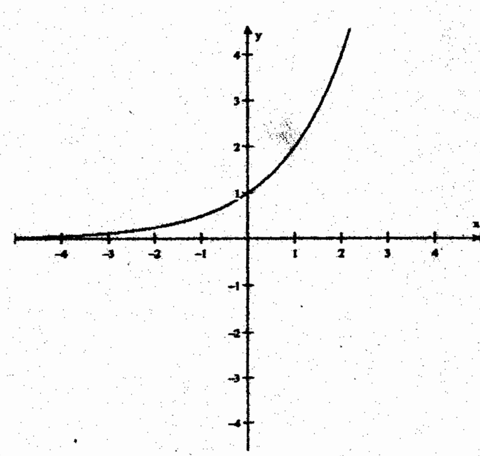
yes

9)



10)

yes



11) Find the inverse of the function.

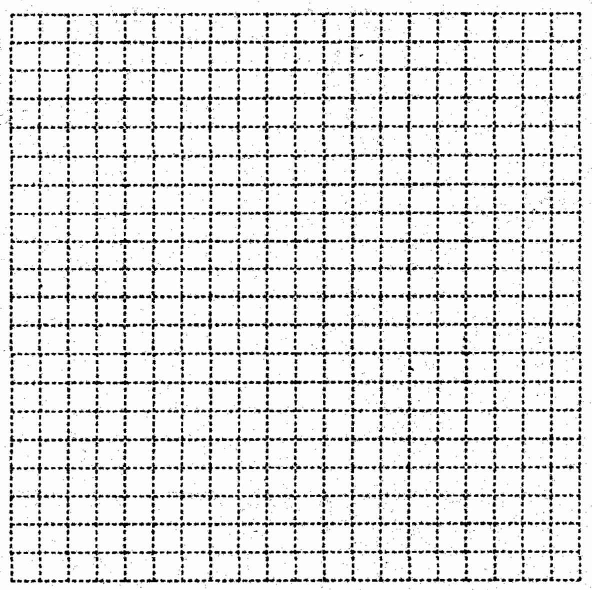
a) $f(x)=2x+3$

b) $f(x)=4x^3-1$

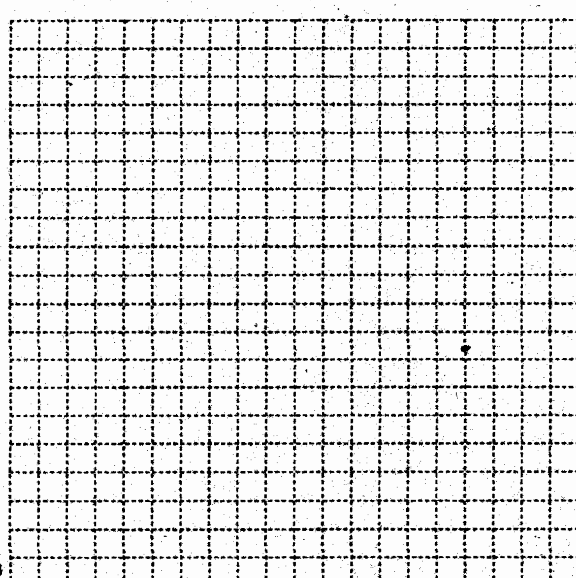
c) $f(x)=\sqrt[3]{5x+7}$

d) $f(x)=\dfrac{8}{2x-3}$

12) Graph the function and its inverse on the same grid.



a) $f(x)=2x-5$



b) $f(x)=(x+5)^3$

The result we just got is strange and unusual!

1) $(f \circ g)(x)$ and $(g \circ f)(x)$ are usually different; but with this $f(x)$ and this $g(x)$, we get the same result, x.

2) $(f \circ g)(x)$ and $(g \circ f)(x)$ are usually more complicated expressions, yet this time we get a very simple result, x.

3) When we put in a value of x, like $x = 23$, we see that $f(23)$ does something to change 23 to a new result, 49,

But... $g(49)$ un-does that, to go back to 23.

Two functions that have this relationship

$$f(g(x)) = x$$
$$g(f(x)) = x$$

are called inverses of each other, or inverse functions.

In particular,

$f(x)$ is the inverse of $g(x)$

$g(x)$ is the inverse of $f(x)$

Notation for this special inverse relationship:

$f(x) = g^{-1}(x)$ "g-inverse-of x"

$g(x) = f^{-1}(x)$ "f-inverse-of x"

CAUTION! This is not an exponent! Not $[f(x)]^{-1} = \frac{1}{f(x)}$.

2) Complete the tables for $f(x) = 2x+3$ and $g(x) = \frac{1}{2}(x-3)$

$f^{-1}(x) = g(x)$

x	y=f(x)
-2	−1
0	3
1	5
52	107
-1	1
3	9
5	13
107	217

(−2, −1)

other ordered pairs on f(x):
(−2.5, −2)
(−1.5, 0)
(24.5, 52)

x	y=~~f(x)~~
-1	−2
3	0
5	1
107	52
-2	−2.5
0	−1.5
1	−1
52	24.5

(−1, −2)

These ordered pairs just swap x ↔ y.

other ordered pairs on $f^{-1}(x)$
(9, 3)
(13, 5)
(217, 107)

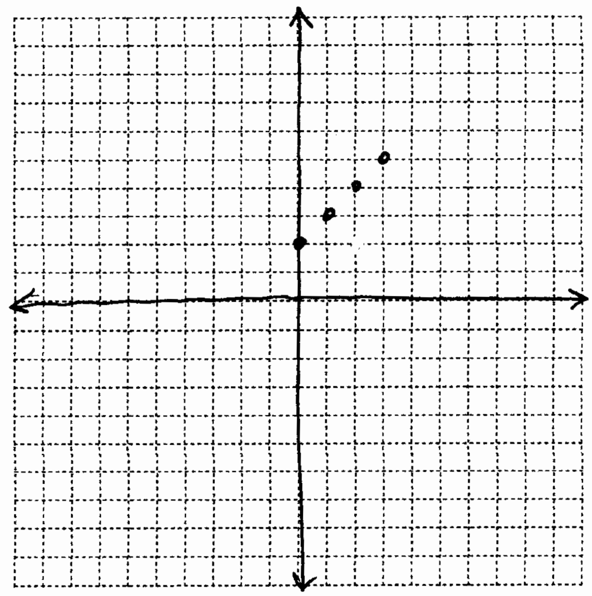
3) Find the inverse of the function. Graph the points of the function. Is the inverse a function?

x	y=f(x)
0	2
1	3
2	4
3	5

swap the locations of x and y.

x	y
2	0
3	1
4	2
5	3

Yes, the inverse is a function.



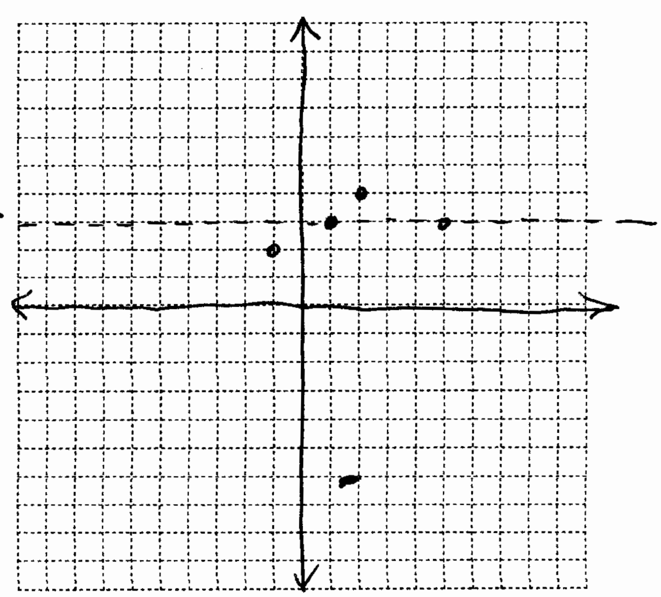
4) Find the inverse of the function. Graph the points of the function. Is the inverse a function?

x	y=f(x)
-1	2
1	3
2	4
5	3

If we can draw any horizontal line that crosses the graph of f more than once, the inverse will not be a function.

x	y
2	-1
3	1
4	2
3	5

no, the inverse is not a function because x=3 has two y-values, 1 and 5



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Horizontal Line Test (H.L.T.)

A function passes the horizontal line test if:
every possible imaginary horizontal line crosses the graph at at most one point.

If a graph passes the horizontal line test, we say that it is one-to-one.

If a graph passes the vertical line test, we say that it is a function.

If a graph passes both the H.L.T. and the V.L.T., we say that it is an invertible function.

f passes VLT $\Rightarrow$ f is a function $\rightarrow$

f passes H.L.T $\Rightarrow$ f is one-to-one $\Rightarrow$ f's inverse is a function $\rightarrow$

if f passes both V.L.T. and H.L.T, it is an invertible function.

While it is always possible to swap the x and y coordinates to get an inverse,
if the result we get by doing so is not a function,
we say it is not an invertible function.

For each graph, identify

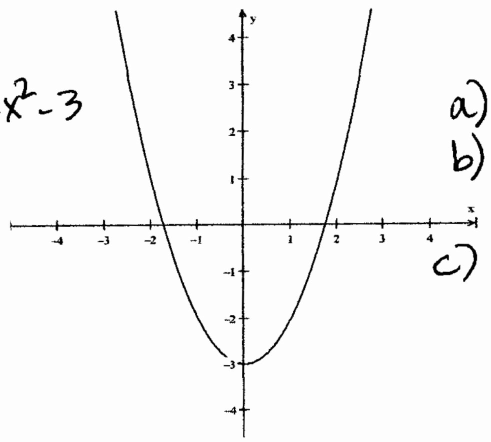
a) Is it a function? = Does it pass the VLT?

b) Is it one-to-one? = Does it pass the HLT?

c) Is it an invertible function? = Does it pass both the VLT and the HLT?

5)

$y=x^2-3$



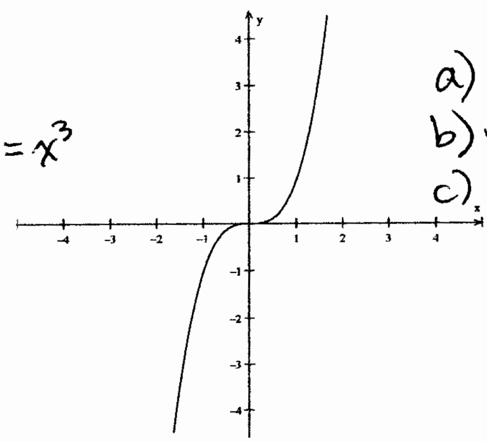
a) yes VLT

b) no HLT

c) no

6)

$y=x^3$



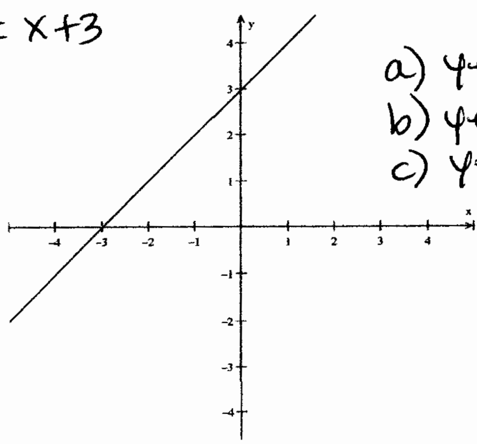
a) yes -VLT

b) yes HLT

c) yes.

7)

$y=x+3$



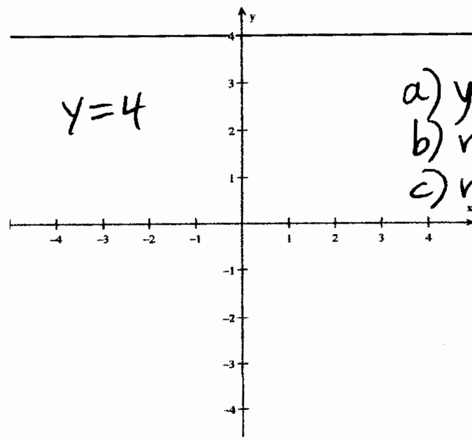
a) yes

b) yes

c) yes

8)

$y=4$



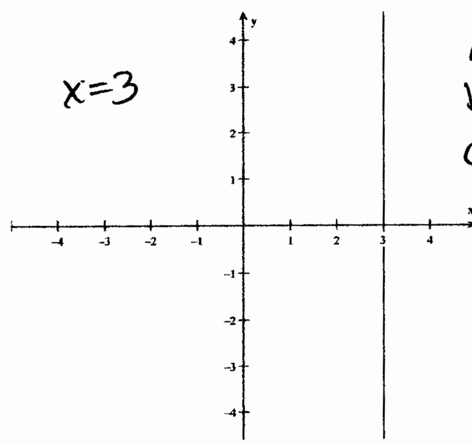
a) yes

b) no HLT

c) no

9)

$x=3$



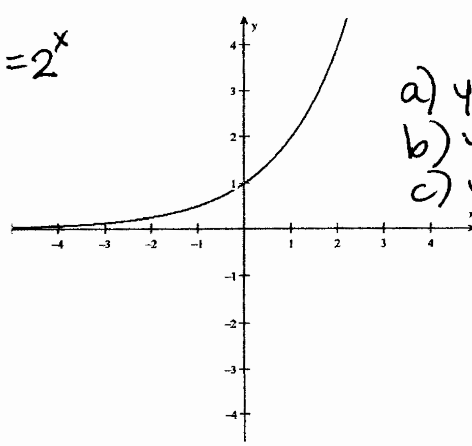
a) no-VLT

b) yes

c) no.

10)

$y=2^x$



a) yes

b) yes

c) yes

11) Find the inverse of the function.

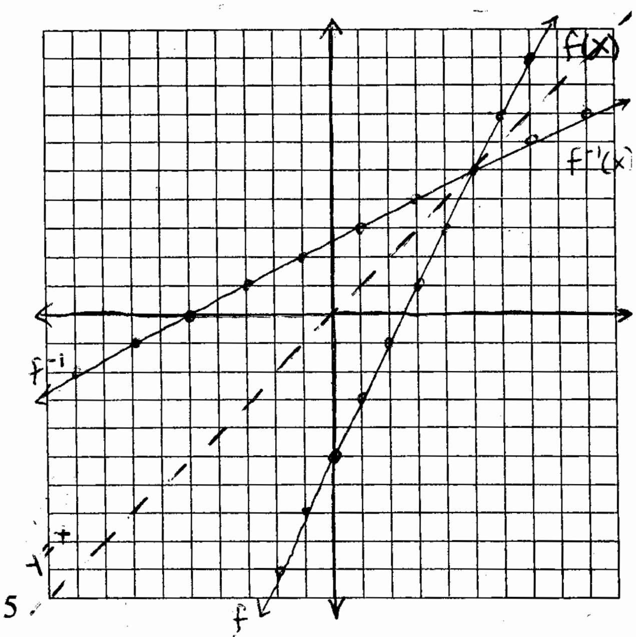
a) $f(x) = 2x + 3$ see next page

b) $f(x) = 4x^3 - 1$

c) $f(x) = \sqrt[3]{5x+7}$

d) $f(x) = \dfrac{8}{2x-3}$

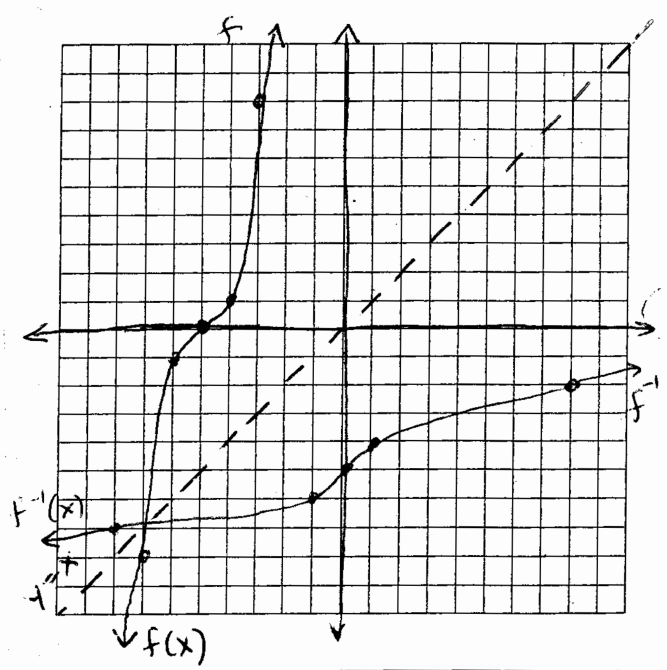
12) Graph the function and its inverse on the same grid.



a) $f(x) = 2x - 5$

Notice that swapping the x and y coordinates creates a graph that is a reflection of the original graph.

This reflection is always a mirror image across the diagonal line y=x.



b) $f(x) = (x+5)^3$

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11 a) $f(x) = 2x+3$

To find inverse:

Step 1: Replace $f(x)$ by y.

step 2: Swap x & y

step 3: Isolate y

step 4: Replace y by $f^{-1}(x)$.

step 1 $y = 2x+3$

step 2 $x = 2y+3$

step 3 $x-3 = 2y$

$$\frac{x-3}{2} = y$$

$$y = \frac{x-3}{2}$$

step 4 $\boxed{f^{-1}(x) = \frac{x-3}{2}}$

b) $f(x) = 4x^3-1$

$y = 4x^3-1$ step 1

$x = 4y^3-1$ step 2

$\frac{x+1}{4} = \frac{4y^3}{4}$ step 3

$$\frac{x+1}{4} = y^3$$

$\sqrt[3]{\frac{x+1}{4}} = y$ To remove exp 3, cube root both sides.

$$\frac{\sqrt[3]{x+1}}{\sqrt[3]{4}} = y$$

$y = \frac{\sqrt[3]{x+1}}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$ ← To rationalize a cube root, need a perfect cube, $\sqrt[3]{2^3} = \sqrt[3]{8} = \sqrt[3]{4} \cdot \sqrt[3]{2}$

$\boxed{f^{-1}(x) = \frac{\sqrt[3]{2(x+1)}}{2}}$ step 4

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c) $f(x) = \sqrt[3]{5x+7}$

step 1 $y = \sqrt[3]{5x+7}$

step 2 $x = \sqrt[3]{5y+7}$ ← To remove a $\sqrt[3]{\ }$, cube both sides.

step 3 $x^3 = 5y+7$

$$\frac{x^3-7}{5} = \frac{5y}{5}$$

$y = \frac{1}{5}(x^3-7)$ or $y = \frac{x^3-7}{5}$

$f^{-1}(x) = \frac{1}{5}(x^3-7)$ $\boxed{f^{-1}(x) = \frac{x^3-7}{5}}$

or step 4 $\boxed{f^{-1}(x) = \frac{1}{5}x^3 - \frac{7}{5}}$

d) $f(x) = \frac{8}{2x-3}$

step 1 $y = \frac{8}{2x-3}$

step 2 $x = \frac{8}{2y-3}$ cross-multiply to clear fractions

$x(2y-3) = 8$ divide by x to both sides

$$2y-3 = \frac{8}{x}$$

$$2y = \frac{8}{x} + 3$$

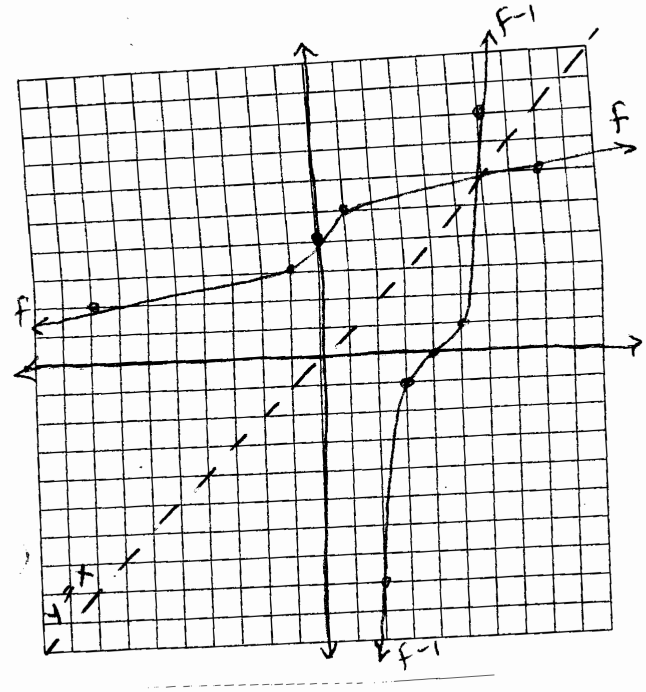
$y = \frac{1}{2}\left(\frac{8}{x}+3\right)$ distribute

$$\boxed{f^{-1}(x) = \frac{4}{x} + \frac{3}{2}}$$

Math 70 Additional Examples

① $f(x) = \sqrt[3]{x} + 4$

$f^{-1}(x) = (x-4)^3$



② $f(x) = \dfrac{3}{x-2}$

$y = \dfrac{3}{x-2}$

$x = \dfrac{3}{y-2}$

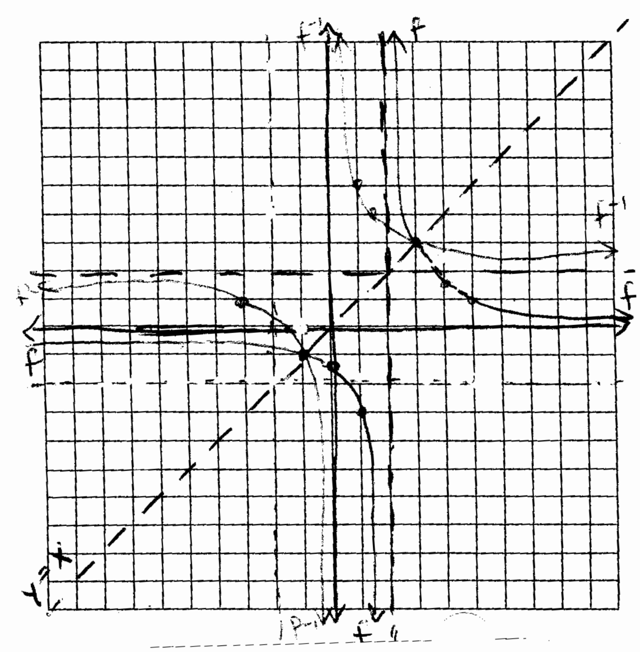
$x(y-2) = 3$

$xy - 2x = 3$

$xy = 2x + 3$

$y = \dfrac{2x+3}{x}$

$f^{-1}(x) = \dfrac{2x+3}{x}$



Math 70 9.2 Inverse Functions

* Practice this even if MathXL can't make you do it! *

(3) Determine if $f(x) = x^3+2$ and $g(x) = \sqrt[3]{x-2}$ are inverses of each other.

Functions which are inverses "un-do" each other → regardless of which function is first.

If $f(x) = x^3+2$ then $f^{-1}(x) = \sqrt[3]{x-2}$

If $f(x) = \sqrt[3]{x-2}$ then $f^{-1}(x) = x^3+2$

To demonstrate that two functions are inverses, must show two things

1) $f(g(x)) = x$

2) $g(f(x)) = x$

x	$f(x)$
3	29
1	3

x	$f^{-1}(x)$
29	3
3	1

$$f(g(x)) = \left(\sqrt[3]{x-2}\right)^3 + 2$$
$$= x - 2 + 2$$
$$= x \quad ✓$$

IMPORTANT: The work is the answer to this type of question

$$g(f(x)) = \sqrt[3]{(x^3+2)-2}$$
$$= \sqrt[3]{x^3}$$
$$= x \quad ✓$$

$f(g(x)) = x$ ← may make more sense if we demonstrate with a value for x.

If $x=1$

$g(1) = \sqrt[3]{1-2} = \sqrt[3]{-1} = -1$

$f(-1) = (-1)^3 + 2 = -1+2 = 1$ ← same value, $x=1$ as at start.

$f(g(1)) = 1$

Similarly for $g(f(x)) = x$:

If $x=1$

$f(1) = 1^3+2 = 3$

$g(3) = \sqrt[3]{3-2} = \sqrt[3]{1} = 1$ ← same value $x=1$ as at start.

$g(f(1)) = 1$

Math 70

9.1.27 If $f(x) = x^2 + 8$, $g(x) = \sqrt{x}$ and $h(x) = 2x$, write $F(x) = 4x^2 + 8$ as a composition using two of the given functions.

$F(x) = (\square \circ \square)(x)$

Given $f(x) = x^2+8$
$g(x) = \sqrt{x}$
$h(x) = 2x$.

Rewrite $F(x) = 4x^2+8$ as a composition of two functions.
Notice: $g(x) = \sqrt{x}$ has a square root, but
$F(x) = 4x^2+8$ has no square root.

So we probably aren't going to use $g(x)$ at all.
We'll use $f(x) = x^2+8$
$h(x) = 2x$.

It's either $(f\circ h)(x)$ or $(h\circ f)(x)$.
Work out what these are:

$(f\circ h)(x)$	$(h\circ f)(x)$
$= f(h(x))$	$= h(f(x))$
$= (2x)^2+8$	$= 2(x^2+8)$
$= 4x^2+8$	$= 2x^2+16$

This is what we wanted.

$\boxed{F(x) = (f\circ h)(x).}$

Extra composition

$(f \circ g)(x) = f(g(x))$ is not the same as $(g \circ f)(x) = g(f(x))$

as we saw in our previous work:

① e) $(g \circ f)(x) = 6x^2 + 8x - 3$

f) $(f \circ g)(x) = 12x^2 - 52x + 56$

② g) $(g \circ f)(x) = 9$

h) $(f \circ g)(x) = 4$

③ On interstate trips, a driver averages 54 mph. The distance d in miles traveled in t hours is given by $d(t) = 54t$. Because the driver averages 25 miles per gallon, the number of gallons g used is given by $g(d) = d/25$. The cost per gallon is \$2.95, so the total fuel cost is given by $c(g) = 2.95g$.

a) Write a function describing the number of gallons used in t hours of travel.
b) Write a function describing the total fuel cost in t hours of travel.
c) Determine the total fuel cost of a 12-hour trip.

a) $g(d(t)) = \frac{54t}{25} = \boxed{2.16t}$

b) $c(g(d(t))) = 2.95(2.16t) = \boxed{6.372t}$

c) $c(g(d(t))) = (6.372)(12) = \$76.464 \approx \boxed{\$76.46}$

④ An oil tanker runs aground and springs a leak. The oil spreads out in a semicircular pattern from the shoreline. The distance r (in feet) from the tanker to the edge of the oil spill at time t (in minutes) is given by the function $r(t) = 20t$.

a) If the area of the semicircle is given by $A(x) = \frac{1}{2}\pi x^2$, where x is the radius, write a function for the area covered by the oil at time t.
b) What is the area of the oil spill after 5 minutes?

a) $A(r(t)) = \frac{1}{2}\pi(20t)^2$

$= \frac{1}{2}\pi \cdot 400t^2$

$= \boxed{200\pi t^2}$

b) $A(r(5)) = 200\pi(5)^2$

$= 200 \cdot 25 \cdot \pi$

$= \boxed{5000\pi \text{ ft}^2}$

$\approx \boxed{15{,}707.96 \text{ ft}^2}$

This is function composition also.

$A(r(t))$ is the method (replace x in $A(x)$ by expression $r(t) = 20t$).

$(A \circ r)(t)$ means "A composed on r of t" and this is the name of the function

5. Given $f(x) = 2x+3$
$g(x) = \frac{1}{2}(x-3)$

a) Find $(g \circ f)(x) = g(f(x))$
$= \frac{1}{2}[(2x+3)-3]$
$= \frac{1}{2}[2x]$

$\boxed{(g \circ f)(x) = x}$

b) Find $(f \circ g)(x) = f(g(x))$
$= 2[\frac{1}{2}(x-3)]+3$
$= x-3+3$

$\boxed{(f \circ g)(x) = x}$

What does this mean, that $f(g(x)) = x$?
Pick a random number –
$x = 7$.

Find $(f \circ g)(7) = f(g(7)) = f(\underbrace{\tfrac{1}{2}(4)}_{\frac{1}{2}(7-3)}) = f(2) = 2(2)+3 = \boxed{7}$

$x = 7$ in

f and g un-do each other

$x = 7$ out

Similarly, $g(f(x)) = x$ for a random value of x –
$x = -23$.

Find $(g \circ f)(-23) = g(f(-23)) = \boxed{-23}$

$x = -23$ in

$x = -23$ out

So $f(x) = 2x+3$ and $g(x) = \frac{1}{2}(x-3)$ have a special relationship to each other because when we compose them, they un-do each other.
This special relationship is the focus of section 9.2.